

INTRODUCTION IN PROBABILITIES

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PROBABILITY THEORY

✕ Probability

- + A numerical measure of uncertainty
- + Quantify the terms “unlikely”, “possible”, “likely”, “probable”, etc.

DEFINITION

- ✗ The experiment: putting in practice the condition for studying different situations. Each time when the experiment is done, the result is random

DEFINITION

- ✗ The outcome is the result of one execution of the experiment
- ✗ The event- the result of an outcome
- ✗ The elementary event – the result of JUST one outcome

THE CLASSICAL DEFINITION OF PROBABILITY

- ✖ Assuming that for an experiment, all the events are equally plausible, the probability for a particular event is equal with the ratio between the number of the possible ways for that particular event to happen and the total number of outcomes that the experiment could generate

$$P(A) = \frac{s}{n}$$

EXAMPLE

- ✖ What is the probability that the head will fall by throwing a coin?
- ✖ Possible outcomes: $n=2$ – (head, tail);
- ✖ favorable cases: $s=1$ (head);
- ✖ probability = $1/2$



PROBLEM:

- ✖ If we random select a person from a community, what is the chance to have the blood type O?

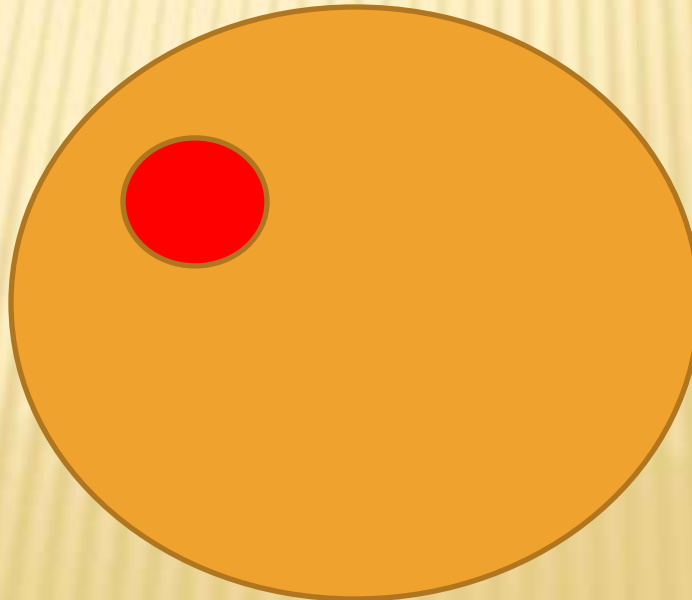
Blood type	Probability
O	0,42
A	0,43
B	0,11
AB	0,04

THE AXIOMATIC DEFINITION FOR PROBABILITY

- ✗ Having an experiment (H) for which E is the set of all possible results. E is called event space.
- ✗ Any subset A of E is called the event. If A contains just one element from E , this event will be called an elementary event
- ✗ Any event whose realization depends on at least two events is a compound event

EVENTS - SETS

- ✗ Event space – the set of all sub-sets
- ✗ Events – sub-sets



THE AXIOMATIC DEFINITION FOR PROBABILITY

- ✗ The null set $\emptyset$ and the event space E are also events, the impossible event and the certain event

COMPOSED EVENTS (OPERATION WITH EVENTS)

- ✗ Union $A \cup B$
- ✗ Intersection $A \cap B$
- ✗ Oposite of A $\text{non } A$
- ✗ Incompatble events
- ✗ Compatible events
- ✗ Involving events

THE AXIOMATIC DEFINITION FOR PROBABILITY

- ✗ Having an event space (E) associated to an experiment H and Ω the set of all events (all parts of E).
- ✗ The function $P:\Omega\rightarrow\mathbb{R}$ is called probability function, and $P(A)$ is called probability of the event A , if the following axioms are satisfied:
 - + $0 \leq P(A) \leq 1$, for any event A of Ω
 - + $P(E)=1$
 - + If A și B are incompatible, then
$$P(A \cup B) = P(A) + P(B)$$

NOTATIONS

✕ For events : {.....}

+ Ex. $A = \{\text{the age of a person is no more than 20 years}\}$

✕ Probability is a number

+ $P(A) = x$

PROPERTIES

✗ If $A_1, A_2, \dots, A_n$ are incompatible events then

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i)$$

- $P(\emptyset)=0$
- $P(\text{non } A) = 1-P(A)$

PROPERTIES

✕ For any events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

PROBABILITY OF THE EVENTS

- ✗ Probability quantifies what SHOULD happen not what WILL happen

INDEPENDENT EVENTS

- ✖ Two events A and B are independent if and only if

$$P(A \cap B) = P(A) \times P(B)$$

DEPENDENT EVENTS

- ✗ Two events A and B are dependent if

$$P(A \cap B) \neq P(A) \times P(B)$$

APPLICATIONS

- ✗ In a study about the familial aggregation of hypertension researchers compute the probability of hypertension on men $P(A)=0,2$ and on women $P(B)=0,1$
- ✗ Compute the probability to have hypertension in the family (at least one person to have hypertension)?

Probability that at least one person in family to have hypertension is the probability that:

- Man has hypertension
- Woman has hypertension
- Both have hypertension
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.1 - 0.2 * 0.1 = 0.28$

APPLICATIONS

- + In a study about the familial aggregation of hypertension researchers compute the probability of hypertension on mother $P(A)=0,1$, on the first child $P(B)=0,2$ and the frequency of hypertension on children having the mother affected by hypertension $\Pr(A \cap B) = 0,5$
- + Are those events independent?

If the events are independent then
 $P(A \cap B) = P(A) \times P(B)$

$$P(A) \times P(B) = 0,1 * 0,2 = 0,02$$

$P(A \cap B) = 0,5$, so the events are not independent

CONDITIONAL PROBABILITIES

- ✖ If A and B are two arbitrary events, the conditional probability of A given B- $P(A|B)$ - is the probability of the event A to occur when event B already have occurred.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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- ✗ If A and B are independent events then
 $P(B | A) = P(B)$

PROBABILITY OF INTERSECTION

✕ For any events A and B

$$P(A \cap B) = P(A) \times P(B|A) = P(B) \times P(A|B)$$

PROBABILITY

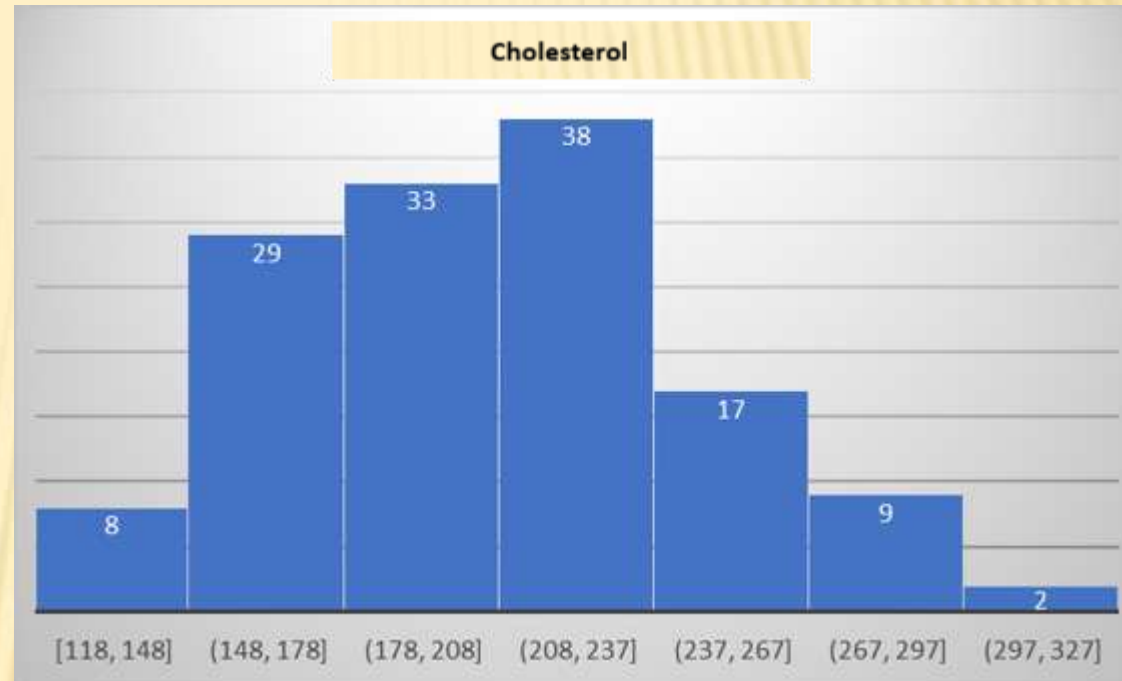
- ✖ The relative frequency in the sample is a probability function!
- ✖ Age: 19, 19, 20, 21, 20, 19, 19, 20, 21, 20
- ✖ $F(19)=4/10$; $F(20)=4/10$; $F(21)=2/10$

PREVALENCE

- ✖ If we have a sample of size n in which we study the presence of a disease, this disease being encountered in k people, the prevalence of the disease in the sample is k / n
- ✖ The probability for random selecting one person from the sample to have the disease is k/n

DESCRIPTIVE STATISTICS - PROBABILITIES

Colesterol (mg/dL)							
212	186	232	215	195	219	210	172
176	267	278	147	262	241	235	226
224	240	256	226	277	168	183	230
164	235	197	230	177	184	183	
234	230	215	197	223	217	211	
252	237	206	156	185	239	179	
172	194	224	189	171	217	244	
184	208	270	199	169	241	243	
143	201	234	275	148	167	168	
166	211	278	201	133	205	174	
227	205	239	163	179	188	258	
259	181	286	172	120	184	215	
155	298	145	176	118	162	239	
229	157	171	136	209	216	191	
286	226	232	210	210	176	150	
267	224	228	327	202	201	229	
158	179	152	281	174	201	255	
187	213	291	174	183	187	194	
185	187	163	223	141	176	245	



Size 136;
Mean 206, StDev. 40
Min 118, Max 327

(135 165 195 220 250 280 310)
(5,88% 21,32% 24,26% 27,94% 12,50% 6,62% 1,47%)

CONTINUOUS DISTRIBUTION

- ✖ For a continuous variable X we can define $f: \mathbb{R} \rightarrow \mathbb{R}$ called density of probability, having the properties:

$$\Pr(a \leq X \leq b) = \int_a^b f(x) dx$$

- ✖ $f(x) \geq 0, \forall x \in \mathbb{R}$ $\int_{-\infty}^{\infty} f(x) dx = 1$

$$F(x) = \Pr(X \leq x) = \int_{-\infty}^x f(t) dt$$

$$M(X) = \int_{-\infty}^{\infty} xf(x) dx$$

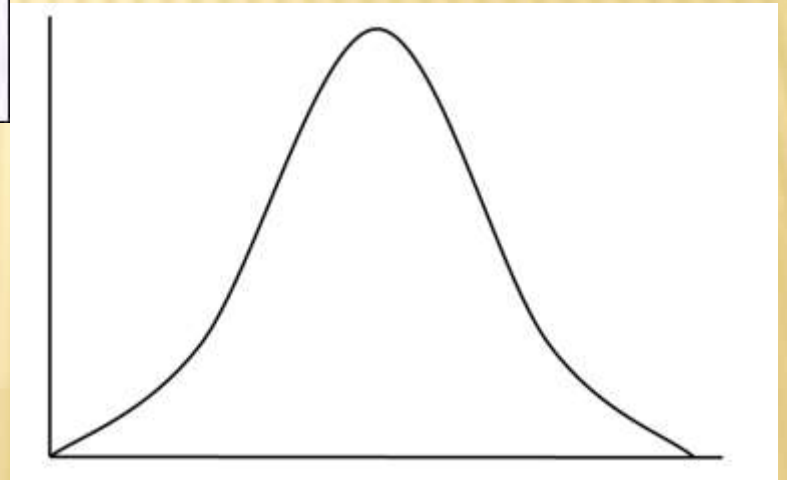
$$V(X) = \int_{-\infty}^{\infty} [x - M(X)]^2 f(x) dx$$

THE MAIN PROBABILITIES DISTRIBUTIONS

In general, the distributions of variables in a population under study are unknown. In practice, attempts are made to fit these distributions to theoretical probability distributions that serve as models for these statistical variables.

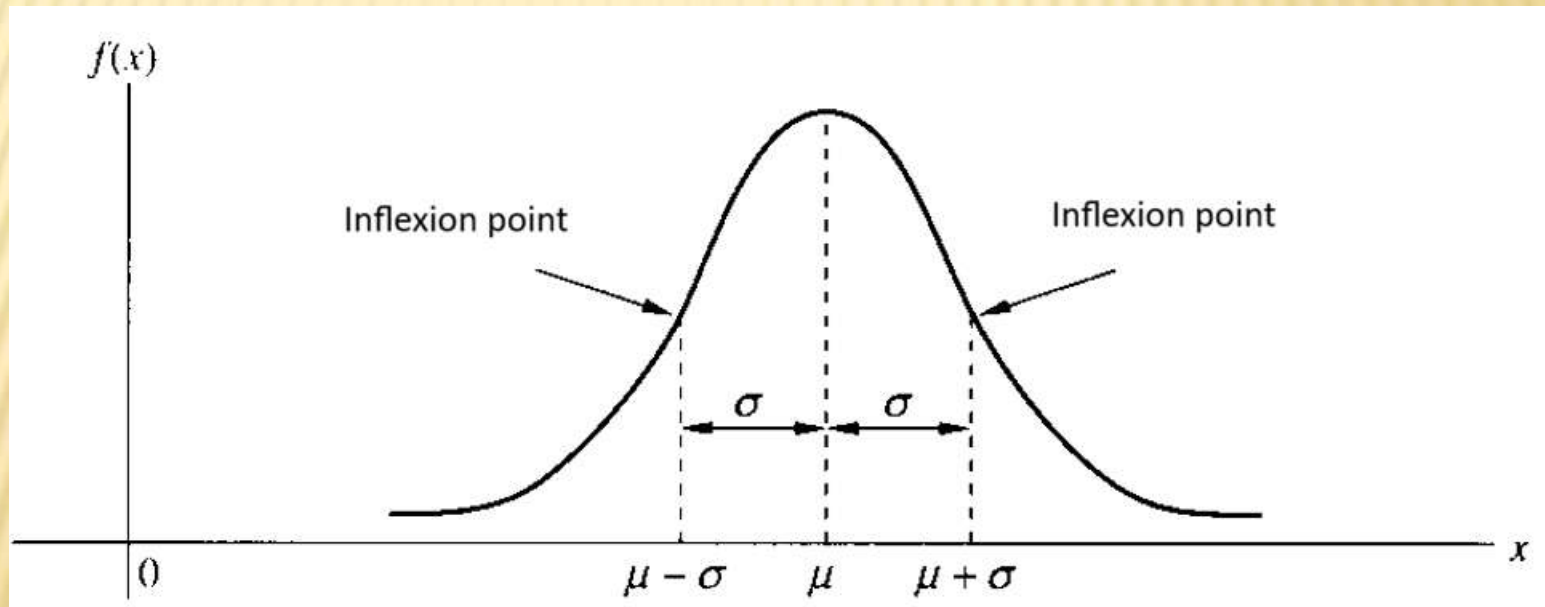
NORMAL DISTRIBUTION

✖ Carl Friedrich Gauss (1777 – 1855)



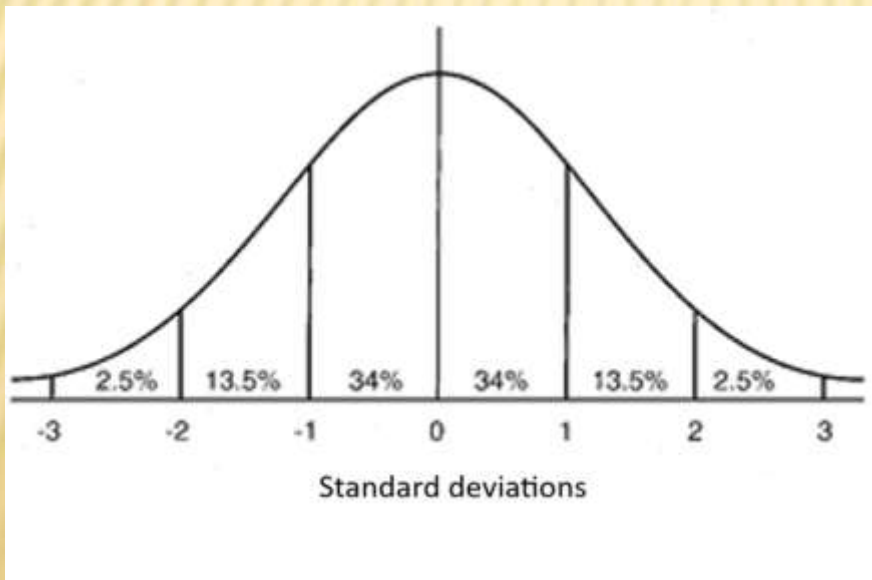
NORMAL DISTRIBUTION

- ✗ Depends by mean and the standard deviation



PROPERTIES

- ✗ The interval $\text{mean} \pm 1$ standard deviation contains approximately 68% of observations.
- ✗ The interval $\text{mean} \pm 2$ standard deviations contains approximately 95% of observations
- ✗ The interval $\text{mean} \pm 3$ standard deviations contains approximately 99.7% of observations.

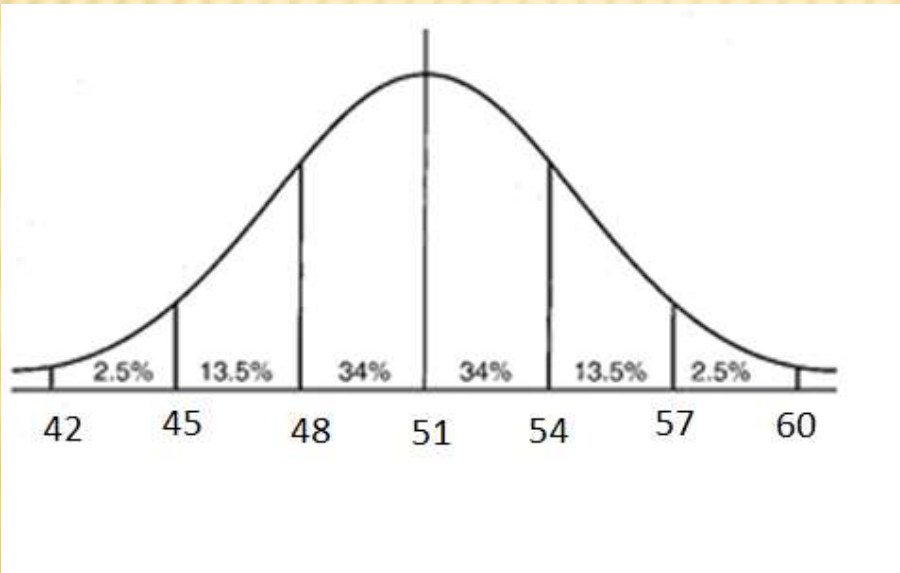


The skewness = 0

The excess of kurtosis = 0

QUESTION

- ✗ Given that, under normal conditions, the mean length of newborn boys is 51 cm, with a standard deviation of 3 cm, what is the probability that a newborn boy will be longer than 57 cm?



R: 2,5%