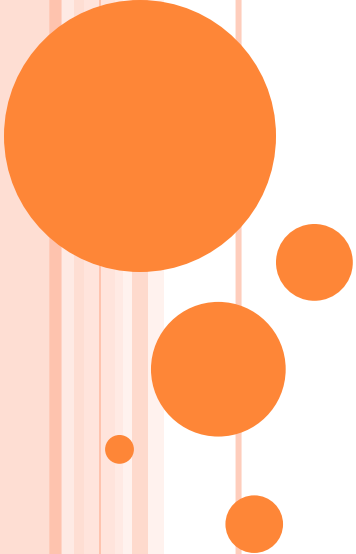


STATISTICAL TESTS - INTRODUCTION



Tudor Calinici
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OBJECTIVES

- Define what are the statistical tests
- Explain how a statistical test is performed
- Define the types of errors in statistical tests
- Define statistical significance
- Present different types of statistical tests
- Know the specific language!



HYPOTHESIS

- The formulation of new hypotheses (or models or theories) is one of the most important aspects of scientific research. These hypotheses are trying to describe or explain real phenomena.



POPULATION - SAMPLE

- When a statistic investigation is carried out on a sample, any result obtained has a relative value.
- We can not even say with certainty what is the difference between the observed data in the sample and the data in population, since the state of the population is usually unknown.



POPULATION - SAMPLE - RESULTS

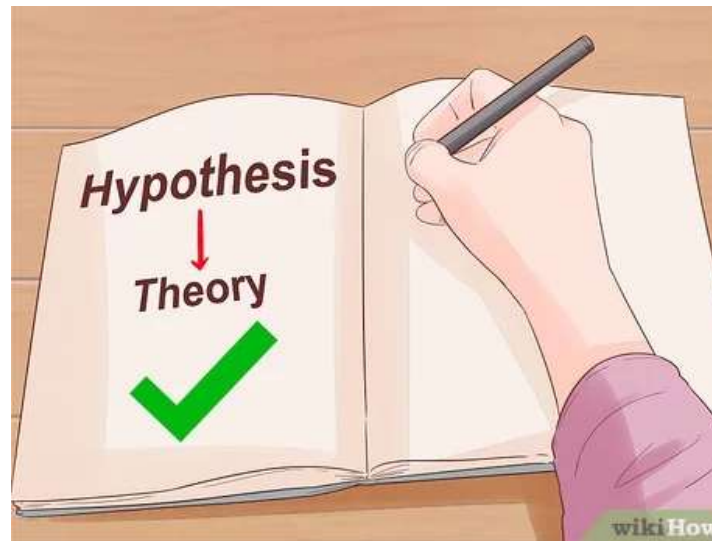


STATISTICAL HYPOTHESIS

- Therefore, referring to the populations studied starting from the samples, the researchers can not state judgments with certainty, but can express some assumptions which are called **statistical hypotheses**.
- **Statistical tests** will evaluate the statistical hypotheses. After the test, the statistical hypotheses will be accepted or rejected.



HYPOTHESIS



- Prolonged exposure to cigarette smoke increases the risk of developing lung cancer.
- The prevalence of lung cancer among smokers should be higher than the prevalence among nonsmokers!



HOW TO TEST A HYPOTHESIS

- Formulating scientific hypothesis
Formulating statistical hypothesis
- Testing
 - Drawing a sample from the target population
 - Checking whether the sample supports the hypothesis
- Repeating the test
- **Test the statistical hypothesis using the appropriate statistical test**



THE MODEL FOR THE STATISTICAL TESTS

- Statistical hypothesis testing is done by comparing two competing hypotheses :
 - Null hypothesis - the model that the researcher wants to replace
 - Alternative hypothesis - the new model to replace the null hypothesis



THE GOAL OF THE STATISTICAL TEST

- The goal of the statistical test is to reject the null hypothesis
- The result of the test:
 - Reject the null hypothesis and accept the alternative one
 - Can't reject the null hypothesis, can't demonstrate the alternative one



THE STEPS OF THE STATISTICAL TEST

- **To define the statistical hypotheses**
- **To choose the right parameter**
- **To choose the level of significance**
- **To choose the critical region**
- **To compute the value of the parameter**
- **To decide the result of the test**



FORMULATION OF PROBLEMS IN TERMS OF STATISTICAL HYPOTHESIS

- Define the null hypothesis - the test is conducted under the assumption that the null hypothesis is true
- Define the alternative hypothesis - also called a working hypothesis – contradicts the null hypothesis





H0 – There is no difference according to the age between people attending Untold Festival and people attending Philharmonics concerts

H1 – People attending Untold festival are younger than people attending Philharmonics concerts



THE PARAMETER

- The statistical parameter of the test expressed in some form the difference between the compared elements
- Is specific for any statistical test type

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$
$$s^2 = \frac{\sum_{i=1}^{n_1} (x_i - \bar{x}_1)^2 + \sum_{j=1}^{n_2} (x_j - \bar{x}_2)^2}{n_1 + n_2 - 2}$$



THE LEVEL OF SIGNIFICANCE

- The level of significance is the size of the risk for errors which the researcher is willing to accept and to assume
- Usually the level of significance is between 1 and 5%
- The confidence level = 1-level of significance





We'll try to demonstrate with 95% of confidence that the people who attend Untold Festival are younger than people who attend Philharmonics concerts



THE CRITICAL REGION

- The critical region is an interval or a union of intervals
- It may vary from test to test
- It depends by the level of the significance
- $(-\infty, -Z_{\alpha}]$ or $[Z_{\alpha}, +\infty)$
- $[H^2_{\alpha}, +\infty)$
- $(-\infty, -T_{\alpha}] \cup [T_{\alpha}, +\infty)$
- $(-\infty, -1,96]$ – for age comparing test



THE CALCULATION OF THE PARAMETER ON THE RESEARCH DATA

Nr Crt	Age Untold	Nr Crt	Age Philharmonics
1	38	1	25
2	32	2	54
3	23	3	23
4	38	4	57
5	29	5	29
6	48	6	33
7	35	7	68
8	32	8	66
9	34	9	33
10	37	10	15
11	31	11	24
12	27	12	55
13	18	13	17
14	21	14	31
15	47	15	50
16	28	16	59
17	53	17	20
18	36	18	59
19	18	19	18
20	19	20	33
21	31	21	48
22	36	22	25
23	35	23	34
24	43	24	51
25	30	25	30
26	46	26	52
27	24	27	53
28	26	28	56
29	33	29	70
30	44	30	32
31	55	31	48
32	44	32	65
33	38	33	16
34	39	34	39
35	39	35	20
36	33		
37	31		
38	47		
39	35		
40	48		
41	49		
42	24		
43	40		
44	26		
45	43		
46	50		

- The researchers will compute the value of the parameter using the research data

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$s^2 = \frac{\sum_{i=1}^{n_1} (x_i - \bar{x}_1)^2 + \sum_{j=1}^{n_2} (x_j - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

$$t = -1,58$$



DECISION WITH CRITICAL REGION

- H_0 will be rejected if the value of the parameter is in the critical region. In this situation, alternative hypothesis is accepted, assuming a risk for error $< \alpha$ – type I error
- If the parameter is outside the critical region, the null hypothesis can not be rejected, assuming a risk for error β – type II error





H0 – There is no difference according to the age between people attending Untold Festival and people attending Philharmonics concerts

H1 – People attending Untold festival are younger than people attending Philharmonics concerts

Significance level = 0,05; Confidence level 95%

CR = $(-\infty, -1,96]$ $t=-1,58$

We cannot demonstrate that people attending Untold festival are younger than people attending Philharmonics concerts



CONCLUSION

- Statistical tests are used for testing the statistical hypotheses
- We have to define the null and alternative hypothesis – the goal of the test is to reject the null hypothesis
- The statistical test does not allow us to accept the null hypotheses – we can or cannot reject it
- By rejecting the null hypothesis, we can accept the alternative one with a specific level of confidence



REMEMBER!

- When you are investigating samples, you must use statistical tests!
- Define the null and the alternative hypotheses
- The null hypothesis starts with “There is NO”
- Don’t worry about the formulas – we have computers!
- You are never allowed to accept the null hypothesis. Only the alternative one could be accepted!
- All the time, the statistical test ends with a conclusion
- The conclusion could be “We reject the null hypothesis and, with 95% of confidence, we accept the alternative one so ...” or “We cannot reject the null hypothesis and we cannot demonstrate that ...”



STATISTICAL TESTS

- Used to test statistical hypotheses
- After applying the test we'll decide if we can reject or we cannot reject the null hypotheses
- We'll define the null and alternative hypotheses, and we'll compute the test parameter
- We'll check if the value of the test parameter is in the critical region



DECISION WITH CRITICAL REGION

- H_0 will be rejected if the value of the parameter is in the critical region. In this situation, alternative hypothesis is accepted, assuming a risk for error $< \alpha$ – type I error
- If the parameter is outside the critical region, the null hypothesis can not be rejected, assuming a risk for error β – type II error



CONCLUSION

- By rejecting the null hypothesis, the researcher states that the observed results are not due to chance = the results are significant
- When the null hypothesis is not rejected, researcher states that the observed differences are due to chance and the results are not significant



STATISTICAL SIGNIFICANCE

- P – Probability
- Theoretical value for the minimal level of significance for which the null hypotheses can be rejected
- It is specific to statistical software



THE MEANING OF P

- $0,01 \leq p \leq 0,05$ – significant results
- $0,001 \leq p < 0,01$ – high significant results
- $p < 0,001$ – very high significant results
- $p > 0,05$ – non-significant results
- $0,05 < p \leq 0,1$ – tendency



THE RESULT OF THE TEST USING THE STATISTICAL SIGNIFICANCE

- The null hypothesis can be rejected if the results are at least significant
- If the result of the test are not significant the null hypothesis can not be rejected



ERRORS

		Truth	
		H_0 True	H_0 False
Result	H_0 rejected	Type I error (α)	Correct
	H_0 not rejected	Correct	Type II error (β)



TYPE I ERROR

- = H_0 is rejected but is true
- We concluded that there are real differences although they are due to chance
- We conclude that a treatment is effective based on a misinterpretation



TYPE II ERROR

- = H_0 is not rejected even that it is false
- We concluded that the observed differences are due to chance even when they arise due to differences between samples
- We could abandon a new treatment or a new research direction
- “Power of the test” = $1 - \beta$



TYPE II ERROR ?



Significance level = 0,05; Confidence level 95%

CR = $(-\infty, -1,96]$ $t=-1,58$ $p=0,06$

We cannot demonstrate that people attending Untold festival are younger than people attending Philharmonics concerts



CONCLUSION

- The decision of the statistical test can be made based on the position of the test parameter relative to the critical region or based on the interpretation of the value p – both have to be the same
- For any decision, there is a possible error (type I or type II)



DESCRIPTIVE STATISTICS PARAMETERS

- Mean
- Median
- Variance, etc.

- Frequency

- Difference
- Relationship
- Independence
- Normality
- Correlation, etc.



QUANTITATIVE PARAMETERS

- We compare the height of people who do performance sports with height of people who do not practice performance sports
- Null hypotheses: there is no difference between the height of people who do performance sports and height of people who do not practice performance sports



ALTERNATIVE HYPOTHESES

- The height of people who practice performance gymnastics is lower than the height of people who do not practice performance sports
- The height of people who practice performance basketball is higher than height of people who do not practice performance sports
- The height of people who practice performance skiing is different than height of people who do not practice performance sports



ONE TAIL TEST VS, TWO TAIL TEST

- When we are interested in a specific direction of the comparison (lower, higher) – one tail test
- When we just want to see difference – no matter in which direction – two tail test



EXAMPLE – TWO TAIL TEST

- We are interested to assess the performance of an automatic blood pressure monitor, so we'll compare the results of that with the results given by a manual blood pressure monitor.
- The automatic blood pressure monitor is NOT CONFORM if its results vary more than 10% of the results given by a manual blood pressure monitor.



EXAMPLE – ONE TAIL TEST

- We want to assess the efficacy of a new toothpaste comparing with an older product.
- The new toothpaste will be better if the number of dental caries for people which use it is lower than the number of dental caries for people which use the other product.



SAMPLE TYPE

- Independent sample – there is no connection between the objects in the samples
- Dependent (pair) sample – for each object in first sample we have the corresponding one in the second sample



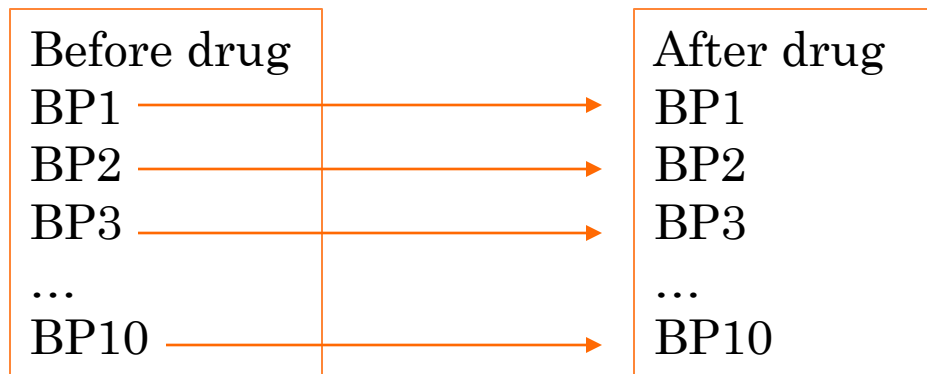
EXAMPLE OF INDEPENDENT SAMPLES

- We want to assess which sport develops better long-distance running ability. For this, we compare the 5 km running time of a group of football players with the 5 km running time of a group of handball players



EXAMPLE OF DEPENDENT SAMPLE

- In order to assess the efficacy of a drug in reducing the blood pressure we measure for 10 persons the blood pressure before and after taking that drug



CONCLUSION

- To apply a statistical test, we must know:
 - What I want to test
 - If it is a quantitative variable -do I use one tail or two tail test
 - What kind of samples are involved (independent, pairs)

